[Reprinted from BULLETIN OF THE AMERICAN METEOROLOGICAL SOCIETY, Vol. 76, No. 1, January 1995]
Printed in U. S. A.

Standardized Test to Evaluate Numerical Weather Prediction Algorithms

Standardized Test to Evaluate Numerical Weather Prediction Algorithms

R. A. Pielke,* L. R. Bernardet,* P. J. Fitzpatrick,* R. F. Hertenstein,* A. S. Jones,* X. Lin,* J. E. Nachamkin,* U. S. Nair,+ J. M. Papineau,* G. S. Poulos,** M. H. Savoie,* and P. L. Vidale*

Abstract

In order to assist in comparing the computational techniques used in different models, the authors propose a standardized set of one-dimensional numerical experiments that could be completed for each model. The results of these experiments, with a simplified form of the computational representation for advection, diffusion, pressure gradient term, Coriolis term, and filter used in the models, should be reported in the peer-reviewed literature. Specific recommendations are described in this paper.

As part of a recent upper-level graduate class on mesoscale meteorological modeling at Colorado State University, students were tasked with evaluating the numerical schemes used by operational and research models on a variety of spatial scales. To perform this assessment, the advection, diffusion, pressure gradient, Coriolis, and filtering terms used by these models were to be reduced to their simplest possible form. Thus, each term was to be reduced to a one-dimensional time-dependent problem in which effects such as grid stagger and multidimensions were ignored.

Much to our surprise, this apparently simple exercise was generally very difficult to do because of the poor documentation in the peer-reviewed literature of the algorithms used by many of the models. While the one-dimensional evaluations are not a complete assessment of the fidelity of a numerical scheme as applied in a model because grid stagger, etc., are not considered, algorithms to represent such effects as advection, diffusion, the pressure gradient force, and the Coriolis term, in their simplest form, should be accurate in terms of how well they preserve both amplitude λ and phase speed c_ϕ. Also, any additional filtering used by a model should be evaluated in terms of its damping effect per application. Such filtering is often used in time and space, as either an explicit filter or a smoothing function.

There are several modelers who do provide effective descriptions of their numerical schemes. These include, for example, Schlesinger (1985, 1988) and Ikawa and Saito (1991). Black (1988, 1994) reports the numerical schemes used in the National Meteorological Center (NMC) eta model, and Janjic (1979, 1984, 1990, 1994) provides additional numerical analyses of that model.

To standardize the level of accuracy of the numerical algorithms, however, it would be valuable to define a set of one-dimensional problems for each modeling group to perform and to publish as part of one of the modeling group's future papers. Once such a study is completed, a firm foundation upon which to base interpretation of different model results will have been formed. To simplify the analyses, the comparisons should be made using cyclic boundary conditions with a sufficient integer number of grid points to include at least one wavelength of the dependent variable(s). The introduction of noncyclic lateral boundary conditions will change the solutions, generally introducing different wavelengths. This addition would complicate the analysis of the solution technique. Our assertion (substantiated by tests) is that the inaccuracies of a numerical scheme for the idealized situation of cyclic boundary conditions are likely to be made only worse when noncyclic lateral boundaries are used.

We propose the following problems to assess the numerical schemes. Each model would discretize these equations in the form as applied in their model.

*Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado.
+Department of Meteorology, IAS/South Dakota School of Mines, Rapid City, South Dakota.
**Los Alamos National Laboratory, Los Alamos, New Mexico.
Corresponding author address: Dr. Roger A. Pielke, Dept. of Atmospheric Science, Colorado State University, Fort Collins, CO 80523.
In final form 3 October 1994.

1) Advection of velocity:

$$\partial u/\partial t = -U\partial u/\partial x, \qquad (1)$$

with U = constant and $u(x, 0) = u_0 \cos kj\Delta x$ ($u_0 = 1$ is usually specified) where $k = 2\pi/n\Delta x$, j is the gridpoint number, Δx is the grid interval, and $n\Delta x$ is the wavelength of the imposed wave. The exact solution is $u_e(x, t) = u(x - Ut)$.

2) Advection of a scalar:

$$\frac{\partial \phi}{\partial t} = -U\partial\phi / \partial x \qquad (2)$$

with U = constant and $\phi(x, 0) = \phi_0 \cos kj\Delta x$ ($\phi_0 = 1$ is usually applied). The exact solution is $\phi_e(x, t) = \phi(x - Ut)$.

The assumed form of solution described by a cosine function does not permit the evaluation of positive definite scalars (i.e., those as kinetic energy, water vapor mixing ratio, etc.), which must always be greater than or equal to zero. An example of a positive definite scheme is that of Smolarkiewicz (1983). The evaluation of a solution technique's ability to preserve positive definiteness is an additional evaluation requirement, although we do not represent a specific methodology in this paper. For most dependent variables, one can work with perturbations from a basic state (e.g., background water vapor mixing ratio, large-scale kinetic energy) so that the analysis procedure presented here still applies. For quantities such as air pollution contaminants, which often are nonzero only locally, positive definiteness is an additional requirement that must be evaluated.

3) Pressure gradient force:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}, \qquad (3)$$

with g (the gravitational constant) and H (a height) constants, and $h(x, 0) = h_0 \cos kj\Delta x$. A value of $h_0 = 1$ can be assumed without loss of generality; $u(x, 0) = (g/H)^{1/2} h(x, 0)$ (Weidman and Pielke 1983). The exact solution is $h_e(x, t) = h(x - ct)$, where $c = (gH)^{1/2}$.

4) Coriolis term:

$$\frac{\partial u}{\partial t} = fv \qquad (4)$$

$$\frac{\partial v}{\partial t} = -fu$$

with f equal to a constant of order 10^{-4} s^{-1}, and $u(x, 0) = u_0 \cos kj\Delta x$ ($u_0 = 1$ can be assumed); $v(x, 0) = 0$. The exact solution of (4) is $u(x, t) = \mathrm{Re}\,[u(x, 0)e^{ift}]$; $v(x, t) = \mathrm{Im}\,[u(x, 0)e^{ift}]$.

5) Diffusion:

$$\frac{\partial \phi}{\partial t} = K\frac{\partial^2 \phi}{\partial x^2} \qquad (5)$$

with K = constant and $\phi(x, 0) = \phi_0 \cos kj\Delta x$ ($\phi_0 = 1$ can be applied) with ϕ_0 a constant. The exact solution of (5) is $\phi_e(x, t) = \exp(-Kk^2 t) \cos kx$.

Any spatial filtering used by a model should be written in a form equivalent to (5). This includes higher-order spatial filtering forms [i.e., fourth order; Klemp and Wilhelmson (1978)]. The filter should then be evaluated as part of the model analysis.

The procedure applied in Pielke (1984, chapter 10) could be used to assess the schemes where values of the change per time step (or per application in the case of a filter) of the dependent variable would be determined. For a leapfrog, centered, advection numerical representation of Eq. (1), for example,

$$\frac{u_j^{\tau+1} - u_j^{\tau-1}}{2\Delta t} = -u\frac{u_{j+1}^{\tau} - u_{j-1}^{\tau}}{2\Delta x}. \qquad (6)$$

This scheme is analyzed in Pielke (1984, p. 280). It exactly preserves amplitude, while phase speed errors are presented as a function of resolved wavelength with features less than about $4\Delta x$ poorly represented in terms of propagation speed.

We suggest for the system of Eqs. (1)–(5) used in a specific atmospheric modeling system, tables of the ratio of the change of the exact solution per time step (i.e., λ_e) versus the change resulting from the analytic solution to the linear discrete equation (i.e., λ_a) should be presented. In addition, tables of the ratio of the exact phase speed c_e (if it is not zero) versus the analytic phase speed c_a would be useful.

For the set of Eqs. (1)–(5) the exact solutions correspond to (1) $\lambda_e = 1$, $c_e = U$; (2) $\lambda_e = 1$, $c_e = U$; (3) $\lambda_e = 1$, $c_e = (gH)^{1/2}$; (4) $\lambda_e = 1$, $c_e = 0$; and (5) $\lambda_e = \exp(-Kk^2 t)$, $c_e = 0$.

Values of the ratios of amplitudes and phase speeds could then be plotted for a range of wavelengths (i.e., $2\Delta x$, $4\Delta x$, $6\Delta x$, $8\Delta x$, $10\Delta x$, $20\Delta x$) and nondimensional time step ratios [i.e., the Courant number $U\Delta t/\Delta x$ for (1) and (2); the gravity wave Courant number $(gH)^{1/2}\Delta t/\Delta x$

for (3); the value of $\Delta t f$ for (4); and the Fourier number $K\Delta t(\Delta x)^{-2}$ for (5)].

The availability of this information will assist readers in determining the quantitative fidelity of numerical algorithms used in a model with respect to their evaluation of individual terms. Modelers could also extend this analysis to show how (and if) grid staggering and multidimensions improve the accuracy of their chosen schemes. At the current time, little information is generally available to readers of the literature that allows an evaluation of the strengths and weaknesses of the various models.

Pielke and Arritt (1984) point out how our community could also benefit from the standardization of code implementations of the various schemes and parameterizations that are widely referenced and used in current models. In particular, the interchangeability of code implementations would have each numerical scheme "engine" already written in standard format, following, ideally, the guidelines of Kalnay et al. (1989) to assist in validation and comparison.

It would further be helpful if at FTP (file transfer protocol) sites authors could make available validations and/or name list examples of a "do it yourself" run plus standardized "boiled-down" one-dimensional versions that would show clearly what each adopted scheme does. Literature should contain all necessary details (e.g., equations, grids, boundary conditions, filters, coefficient ranges, etc.) to understand the model, plus indicate the FTP site where the code implementation of the analytic version is available. As networking technology improves, the community should also encourage standardized indexed documentation [e.g., in HyperText Markup Language, used on National Center for Supercomputing Applications (NCSA 1995) Mosaic pages].

Acknowledgments. This correspondence resulted from class papers and presentations in a Colorado State University, Department of Atmospheric Science course on meteorological modeling. The manuscript was ably typed by Dallas McDonald and Bryan Critchfield. Partial support for the completion of this paper came from NSF Grant ATM-9306754. Two anonymous referees provided very effective suggestions to finalize the paper and, since we do not know who they are, we thank them here.

References

Black, T. L., 1988: The step-mountain, eta coordinate model: A documentation. 47 pp. [Available from NOAA/NWS/NMC, Development Division, W/NMC2, Room 204, Washington, DC 20233.]

——, 1994: The new NMC mesoscale eta description and forecast examples. *Wea. Forecasting,* **2,** 265–278.

Ikawa, M., and K. Saito, 1991: Description of a nonhydrostatic model developed at the Forecast Research department of the MRI. Tech. Rep. 28, Meteorological Research Institute, Japan.

Janjic, Z. L., 1979: Forward–backward scheme modified to prevent two-grid-interval noise and its application in sigma coordinate models. *Contrib. Atmos. Phys.,* **52,** 69–84.

——, 1984: Nonlinear advection schemes and energy cascade on semi-staggered grids. *Mon. Wea. Rev.,* **112,** 1234–1245.

——, 1990: The step-mountain coordinate: Physical package. *Mon. Wea. Rev.,* **118,** 1429–1443.

——, 1994: The step-mountain eta coordinate model: Further developments of the convection, viscous sublayer, and turbulence closure schemes. *Mon. Wea. Rev.,* **122,** 927–945.

Kalnay, E., M. Kanamitsu, J. Pfaendtner, J. Sela, M. Suarez, J. Stackpole, J. Tucillo, L. Umscheid, and D. Williamson, 1989: Rules for interchange of physical parameterizations. *Bull. Amer. Meteor. Soc.,* **70,** 620–623.

Klemp, J. B., and R. B. Wilhelmson, 1978: Simulations of right- and left-moving storms produced through storm splitting. *J. Atmos. Sci.,* **35,** 1097–1110.

NCSA, cited 1995: A beginner's guide to HTML. [Available online from http://www/ncsa.uiuc.edu/General/Internet/WWW/HTMLPrimer.html.]

Pielke, R. A., 1984: *Mesoscale Meteorological Modeling.* Academic Press, 612 pp.

——, and R. W. Arritt, 1984: A proposal to standardize models. *Bull. Amer. Meteor. Soc.,* **65,** 1082.

Schlesinger, R. E., 1985: Effects of upstream-biased third-order space correction terms on multidimensional Crowley advection schemes. *Mon. Wea. Rev.,* **113,** 1109–1130.

——, 1988: Effects of stratospheric lapse rate on thunderstorm cloud-top structure in a three-dimensional numerical simulation. Part I: Some basic results of comparative experiments. *J. Atmos. Sci.,* **45,** 1555–1570.

Smolarkiewicz, P. K., 1983: A simple positive definite advection scheme with small implicit diffusion. *Mon. Wea. Rev.,* **111,** 479–486.

Weidman, S. T., and R. A. Pielke, 1983: A more accurate method for the numerical solution of nonlinear partial differential equations. *J. Comput. Phys.,* **49,** 342–348.